(a) **(i)** Solve for x:

$$2(4 - 3x) + 12 = 7x - 5(2x - 7).$$

(ii) Verify your answer to **(i)** above.

(b) Solve the simultaneous equations:

$$x + y = 7$$
$$x^2 + y^2 = 25.$$

(a) **(i)** Solve for x:

$$2(4 - 3x) + 12 = 7x - 5(2x - 7).$$

$8 - 6x + 12 = 7x - 10x + 35$

$-15 = 3x$

$x = -5$

(ii) Verify your answer to **(i)** above.

$x = -5$

$2(4 - (-15)) + 12$ $\qquad$ $7(-5) - 5(-10 - 7)$

$38 + 12$ $\qquad$ $-35 + 85$

50 $\qquad$ 50

$[50 = 50]$

(b) Solve the simultaneous equations:

$$x + y = 7$$
$$x^2 + y^2 = 25.$$

$x = 7 - y$

$(7 - y)^2 + y^2 = 25$

$y^2 - 7y + 12 = 0$

$(y - 4)(y - 3) = 0$

$y = 4$	$y = 3$
$x = 7 - 4$	$x = 7 - 3$
$x = 3$	$x = 4$
$(3, 4)$	$(4, 3)$

Question 1 **(25 marks)**

(a) Write 6^{-2} and $81^{\frac{1}{2}}$ without using indices.

$6^{-2} =$

$81^{\frac{1}{2}} =$

(b) Express 2^{24} in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$, correct to three significant figures.

(c) Show that $\dfrac{\left(a\sqrt{a}\right)^3}{a^4}$ simplifies to $\sqrt{a}$.

(d) Solve the equation $49^x = 7^{2+x}$ and verify your answer.

Question 1 **(25 marks)**

(a) Write 6^{-2} and $81^{\frac{1}{2}}$ without using indices.

Given the relation $x^{-1} = \frac{1}{x}$, we have $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

Since $x^{\frac{1}{2}} = \sqrt{x}$, we have $81^{\frac{1}{2}} = \sqrt{81} = \pm 9$

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(b) Express 2^{24} in the form $a \times 10^n$, where $1 \leqslant a < 10$ and $n \in \mathbb{Z}$, correct to three significant figures.

From a calculator, we know that $2^{24} = 16,777,216$ so we can write this as 1.678×10^7 correct to three decimal places.

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(c) Show that $\frac{(a\sqrt{a})^3}{a^4}$ simplifies to $\sqrt{a}$.

Firstly we simplify the top line. Note that, as in part **(a)** we can write $a\sqrt{a} = a \cdot a^{\frac{1}{2}} = a^{\frac{3}{2}}$. Now we have

$$\begin{aligned}
\frac{(a\sqrt{a})^3}{a^4} &= \frac{\left(a^{\frac{3}{2}}\right)^3}{a^4} \\
&= \frac{a^{3\left(\frac{3}{2}\right)}}{a^4} \\
&= \frac{a^{\frac{9}{2}}}{a^4} \qquad (b^x)^y = b^{x \cdot y} \\
&= a^{\frac{9}{2}-4} \qquad \frac{b^x}{b^y} = b^{x-y} \\
&= a^{\frac{1}{2}}
\end{aligned}$$

which is equal to $\sqrt{a}$.

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(d) Solve the equation $49^x = 7^{2+x}$ and verify your answer.

We can rewrite the left hand side of the equation as $49^x = \left(7^2\right)^x = 7^{2x}$ so our equation becomes $7^{2x} = 7^{2+x}$. We equate the indices, getting $2x = 2 + x$ which is true when $x = 2$

To verify this we check that $49^2 = 7^{2+2}$, which is indeed true.

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Question 4 **(25 marks)**

(a) Solve the equation $x^2 - 6x - 23 = 0$. Give your answers in the form $a \pm b\sqrt{2}$, where $a, b \in \mathbb{Z}$.

(b) Solve the simultaneous equations:

$$2r - s = 10$$
$$rs - s^2 = 12.$$

Question 4 **(25 marks)**

(a) Solve the equation $x^2 - 6x - 23 = 0$. Give your answer in the form $a \pm b\sqrt{2}$ where $a, b \in \mathbb{Z}$.

We will use the quadratic formula to solve this equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Leftrightarrow \quad x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-23)}}{2(1)}$$

$$\Leftrightarrow \quad x = \frac{6 \pm \sqrt{36+92}}{2}$$

$$\Leftrightarrow \quad x = \frac{6 \pm \sqrt{128}}{2}$$

$$\Leftrightarrow \quad x = \frac{6 \pm \sqrt{64}\sqrt{2}}{2}$$

$$\Leftrightarrow \quad x = \frac{6 \pm 8\sqrt{2}}{2}$$

$$\Leftrightarrow \quad x = 3 \pm 4\sqrt{2}$$

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(b) Solve the simultaneous equations:

$$\begin{aligned} 2r - s &= 10 \\ rs - s^2 &= 12 \end{aligned}$$

We will need to write one variable in terms of the other from one of the two equations and substitute that value into the second equation. We will write r in terms of s from the first equation, and substitute that value of r into the second equation and then solve for s. The first equation tells us that $2r = 10 + s$ or $r = 5 + \frac{1}{2}s$.

The second equation now reads $\left(5 + \frac{1}{2}s\right)s - s^2 = 12$ which expands to $5s + \frac{1}{2}s^2 - s^2 = 12$, or $5s - \frac{1}{2}s^2 = 12$. We can multiply both sides by 2 and rearrange the equation to get $s^2 - 10s + 24 = 0$. This factorises into $(s-4)(s-6) = 0$, i.e. $s = 4$ or $s = 6$.

We finally use these values to solve for r. If $s = 4$, then $r = 5 + \frac{1}{2}(4) = 7$. If $s = 6$, then $r = 5 + \frac{1}{2}(6) = 8$. Thus our solutions are

$$r = 7, s = 4 \qquad \text{or} \qquad r = 8, s = 6$$

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Question 8 **(50 marks)**

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

(b) Write down two formulas – one for each plant – to represent the heights of the two plants on any given day. State clearly the meaning of any letters used in your formulas.

(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.

(Questions continue overleaf.)

(d) **(i)** From your diagram, write down the point of intersection of the two graphs.

Answer: _____

(ii) Explain what the point of intersection means, with respect to the two plants.
Your answer should refer to the meaning of *both* co-ordinates.

(e) Check your answer to part **(d)(i)** using your formulas from part **(b)**.

(f) The point of intersection can be found either by reading the graph or by using algebra.
State one advantage of finding it using algebra.

(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

Question 8 **(50 marks)**

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

We will refer to the plant which was initially 16 cm as plant A and the other as plant B. In the table below, day 0 will refer to the day that John bought the plants.

Day	0	1	2	3	4	5	6	7
A	16cm	20cm	24cm	28cm	32cm	36cm	40cm	44cm
B	24cm	27.5cm	31cm	34.5cm	38cm	40.5cm	44cm	48.5cm

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(b) Write down two formulas – one for each plant – to represent the heights of the two plants on any given day. State clearly the meaning of any letters used in your formulas.

Let $f(x)$ represent the height in centimetres of plant A, x days after John bought it. Similarly, let $g(x)$ represent the height in centimetres of plant B, x days after John bought it. We know the initial values $f(0) = 16$ and $g(0) = 24$. As we increase x by one, f increases by 4 and g increases by 3.5, so our general formulae are

$$f(x) = 16 + 4x \qquad \text{and} \qquad g(x) = 24 + 3.5x$$

We could also determine these by noting that the graphs will be straight lines. So for f, for example, two points on the line will be $(0, 16)$ and $(1, 20)$. We could then use the formula $(y - y_1) = m(x - x_1)$, where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and solve for $y = f(x)$. A similar approach would give g.

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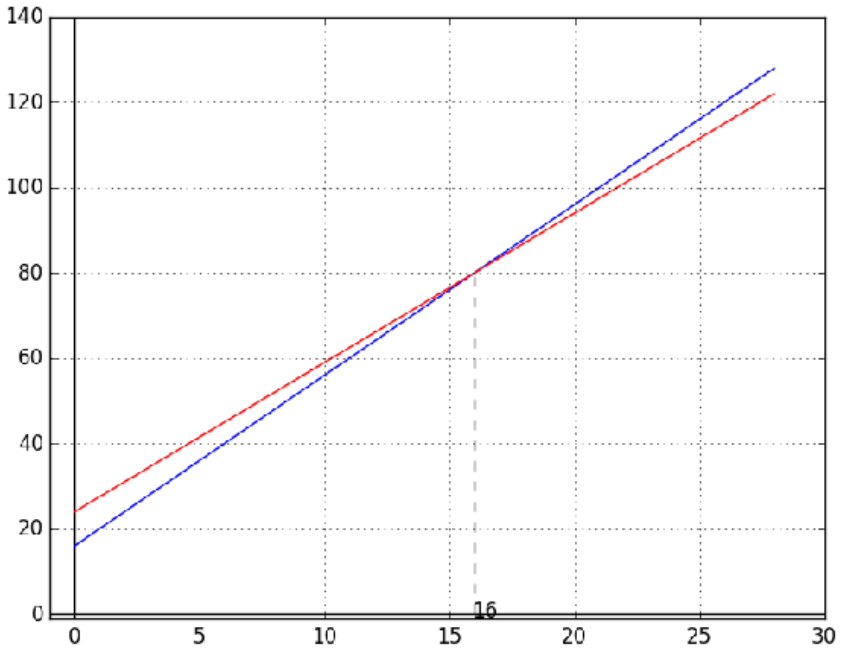
(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.



Here, the blue line represents $f(x)$ (the height of plant A) and the red line represents $g(x)$ (the height of plant B).

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(d) **(i)** From your diagram, write down the point of intersection of the two graphs.

The two graphs intersect when $x = 16$. We can verify this by noting that $f(16) = 16 + 4(16) = 80$ and $g(16) = 24 + 3.5(16) = 80$.

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(ii) Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of *both* co-ordinates.

The point of intersection $(16, 80)$ tells us the day at which the heights of the two plants will be equal. Since the x coordinate represents the day and the y coordinate represents the height in centimetres of each plant, on day 16, both plants will be 80 cm tall.

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(e) Check your answer to part **(d)(i)** using your formulas from part **(b)**.

We wish to determine the point x for which $f(x) = g(x)$, i.e.

$$16 + 4x = 24 + 3.5x \quad \Leftrightarrow \quad 0.5x = 8 \quad \Leftrightarrow \quad x = 16$$

as expected.

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(f) The point of intersection can be found either by reading the graph or by using algebra. State one advantage of finding it using algebra.

By using algebra, we can find the point of intersection exactly with no errors. When reading from a graph, the point has to be estimated, which can introduce errors.

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(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

This model assumes that both plants grow at constant rates. This may not be practically true, as plants may grow at varying speeds. Thus, the graphs for the functions may realistically not be straight lines, which would affect where the point of intersection lies.

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Question 4 **(25 marks)**

(a) Solve the equation $x^2 - 6x - 23 = 0$, giving your answer in the form $a \pm b\sqrt{2}$, where $a, b \in \mathbb{Z}$.

(b) Solve the simultaneous equations

$$2r - s = 10$$

$$rs - s^2 = 12$$

Question 9 **(50 marks)**

(a) A farmer is growing winter wheat. The amount of wheat he will get per hectare depends on, among other things, the amount of nitrogen fertiliser that he uses. For his particular farm, the amount of wheat depends on the nitrogen in the following way:

$$Y = 7000 + 32N - 0{\cdot}1N^2$$

where Y is the amount of wheat produced, in kg per hectare, and N is the amount of nitrogen added, in kg per hectare.

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(i) How much wheat will he get per hectare if he uses 100 kg of nitrogen per hectare?

(ii) Find the amount of nitrogen that he must use in order to maximise the amount of wheat produced.

(iii) What is the maximum possible amount of wheat produced per hectare?

(iv) The farmer's total costs for producing the wheat are €1300 per hectare. He can sell the wheat for €160 per tonne. He can also get €75 per hectare for the leftover straw. If he achieves the maximum amount of wheat, what is his profit per hectare?

(b) A marble is dropped from the top of a fifteen-story building. The height of the marble above the ground, in metres, after t seconds is given by the formula:

$$h(t) = 44{\cdot}1 - 4{\cdot}9t^2 .$$

Find the speed at which the marble hits the ground.

Give your answer **(i)** in metres per second, and

(ii) in kilometres per hour.

(a) Solve the equation $\frac{1}{2}(7x-2)+5=2x+7$.

(b) Solve the equation $\frac{2}{3x-4}-\frac{1}{2x+1}=\frac{1}{2}$ and give your answers correct to one decimal place.

$$\tfrac{1}{2}(7x-2)+5=2x+7$$
$$\Rightarrow 7x-2+10=4x+14$$
$$\Rightarrow 7x+8=4x+14$$
$$\Rightarrow 3x=6$$
$$\Rightarrow x=2$$

$$\frac{2}{3x-4}-\frac{1}{2x+1}=\frac{1}{2}$$
$$\Rightarrow \frac{2(2x+1)(2)-1(3x-4)(2)=1(3x-4)(2x+1)}{(3x-4)(2x+1)(2)}$$
$$\Rightarrow 8x+4-6x+8=6x^2+3x-8x-4$$
$$\Rightarrow 6x^2-7x-16=0$$
$$\Rightarrow x=\frac{7\pm\sqrt{(-7)^2-4(6)(-16)}}{2(6)}=\frac{7\pm\sqrt{433}}{12}$$
$$\Rightarrow x=2{\cdot}3 \text{ or } x=-1{\cdot}2$$