Question 1

Let x be the hours that Mark worked in Bob's Bakery, and let y be the hours that Mark worked in Ciara's Café. We know that the total hours worked in a week is 34, so $x+y=34$. Given the rates of pay from the two establishments and the total amount earned, we know that $(11.5)x+(9.3)y=362.4$

We can solve these equations simultaneously:

$$\begin{aligned}(11.5)x+(9.3)y &= 362.4\\ x+y &= 34\end{aligned}$$

Since we want to find the number of hours worked in Bob's Bakery, i.e. x, we will eliminate y. We multiply the second equation by 9.3 and subtract:

$$\begin{array}{rcl}(11.5)x+(9.3)y &=& 362.4\\ (9.3)x+(9.3)y &=& 316.2\\ \hline 2.2x &=& 46.2\end{array}$$

Thus $x=\frac{46.2}{2.2}=21$ and so Mark worked 21 hours in Bob's Bakery.

Question 2

(i) Write down each of these new numbers, in terms of x.

Increase x by 1:	$x+1$
Decrease x by 2:	$x-2$

(ii) The product of these two new numbers is 1.
Use this information to write an equation in x.

$(x+1)(x-2)=1$ *or equivalent.*

(iii) Solve this equation to find the two possible values of x.
Give each of your answers correct to 3 decimal places.

$$(x+1)(x-2)=1$$
$$\Rightarrow \quad x^2-x-3=0$$
$$\Rightarrow \quad x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$$
$$\Rightarrow \quad x=2\cdot 3028\ldots \quad \text{and} \quad x=-1\cdot 3028\ldots$$
$$\Rightarrow \quad x=2\cdot 303 \quad \text{and} \quad x=-1\cdot 303,$$ correct to three decimal places.

Question 3

Calculate how much it costs for a car and for a small van to go through the M50 Toll.

$$5x+4y=30-7\cdot 90=22\cdot 10$$
$$2x+6y=30-8\cdot 0=21\cdot 60$$
$$\Rightarrow x=€2\cdot 10$$
$$y=€2\cdot 90$$

Question 4

(a) Find the amount of energy stored in a capacitor when $C=2500$ and $V=32$.

$$W=\frac{1}{2}CV^2$$
$$W=\frac{1}{2}(2500)(32)^2$$
$$W\ 1\ 280\ 000$$

(b) Write V in terms of W and C.

$$W = \frac{1}{2}CV^2$$

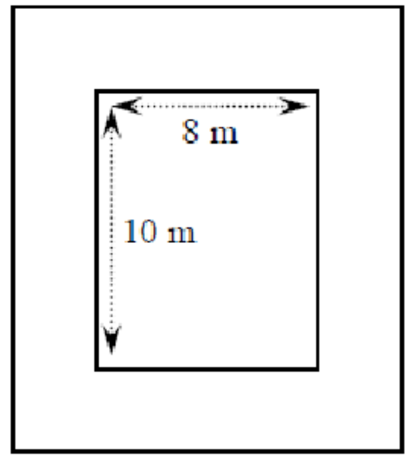
$$2W = CV^2$$

$$\frac{2W}{C} = V^2$$

$$\sqrt{\frac{2W}{C}} = V$$

Question 9 **(Suggested maximum time: 20 minutes)**

A plot consists of a rectangular garden measuring 8 m by 10 m, surrounded by a path of constant width, as shown in the diagram. The total area of the plot (garden and path) is 143 m^2.



Three students, Kevin, Elaine, and Tony, have been given the problem of trying to find the width of the path. Each of them is using a different method, but all of them are using x to represent the width of the path.

Kevin divides the path into eight pieces. He writes down the area of each piece in terms of x. He then forms an equation by setting the area of the path plus the area of the garden equal to the total area of the plot.

(a) Write, in terms of x, the area of each section into Kevin's diagram below.

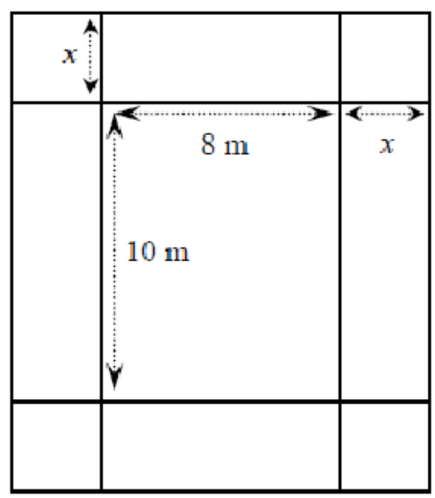
We have four different types of sections in Kevin's diagram: the four corner squares, the rectangles at the top and bottom, the rectangles at the sides and the rectangle in the middle. the areas are given as follows:

Corners: The area of these squares is $x \times x = x^2$ m^2

Top and Bottom: The area of these rectangles is $x \times 8 = 8x$ m^2

Sides: The area of these rectangles is $x \times 10 = 10x$ m^2

Centre: The area of this rectangle is $8 \times 10 = 80$ m^2



Kevin's Diagram

(b) Write down and simplify the equation that Kevin should get. Give your answer in the form $ax^2 + bx + c = 0$.

If we add the areas from all of Kevin's sections, we will get the total area, which is 143 m^2.

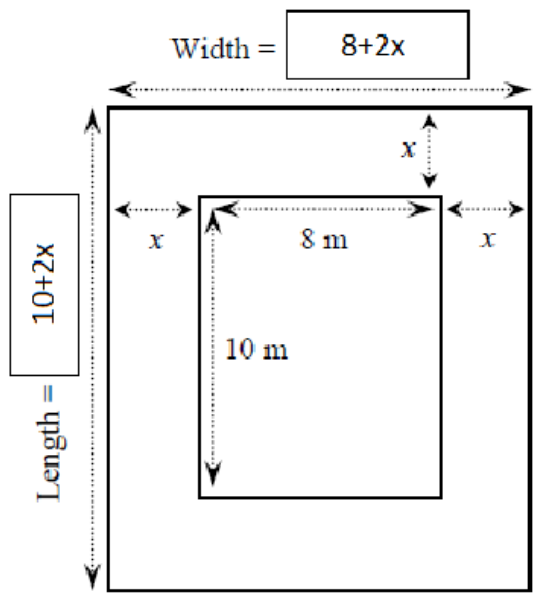
$$4(x^2)+2(8x)+2(10x)+1(80)=143 \quad\Leftrightarrow\quad 4x^2+16x+20x+80=143$$
$$\Leftrightarrow\quad 4x^2+36x-63=0$$

Elaine writes down the length and width of the plot in terms of x. She multiplies these and sets the answer equal to the total area of the plot.

(c) Write, in terms of x, the length and the width of the plot in the spaces on Elaine's diagram.

The overall width of the plot consists of the width of the garden, plus the width of the path on both sides, so $8+2x$.

Similarly, the overall length of the plot consists of the length of the garden, plus the width of the path on both sides, so $10+2x$.



Elaine's Diagram

(d) Write down and simplify the equation that Elaine should get. Give your answer in the form $ax^2+bx+c=0$.

The overall width times the overall length will give the total area of the plot. So:

$$\begin{aligned}(8+2x)(10+2x) = 143 \quad &\Leftrightarrow \quad 80+16x+20x+4x^2 = 143\\ &\Leftrightarrow \quad 4x^2+36x-63 = 0\end{aligned}$$

As we might have expected, this agrees with Kevin's calculations.

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(e) Solve an equation to find the width of the path.

We can factorise our equation as follows:

$$\begin{aligned}4x^2+36x-63=0 \quad &\Leftrightarrow \quad (2x-3)(2x+21)=0\\ &\Leftrightarrow \quad 2x-3=0 \quad \text{or} \quad 2x+21=0\end{aligned}$$

Thus, either $x=\frac{3}{2}$ or $x=-\frac{21}{2}$. Since our path has to have a positive width, we take $x=1.5$ m as the width of the path.

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(f) Tony does not answer the problem by solving an equation. Instead, he does it by trying out different values for x. Show some calculations that Tony might have used to solve the problem.

We will test some values for x. The simplest approach is to take $x = 1, 2, 3, \ldots$ until we find two points close the correct area.

$$\begin{aligned} x = 1 \quad &\Rightarrow \quad \text{Area } = (8+2)(10+2) = 120 \text{ m}^2 \\ x = 2 \quad &\Rightarrow \quad \text{Area } = (8+4)(10+4) = 168 \text{ m}^2 \\ x = 3 \quad &\Rightarrow \quad \text{Area } = (8+6)(10+6) = 224 \text{ m}^2 \end{aligned}$$

Since the total area is actually 143, the correct value of x should be between 1 and 2 because $120 < 143 < 168$. We can try $x = 1.5$ as a guess (and in fact we already know this is the correct answer).

$$x = 1.5 \quad \Rightarrow \quad \text{Area } = (8+3)(10+3) = 143 \text{ m}^2$$

(g) Which of the three methods do you think is best? Give a reason for your answer.

Answer: Elaine's method is best.

Reason: Although all three methods give the correct answer, Elaine's method is the quickest and simplest. Tony's method involves estimating the correct answer, which could give rise to some errors. Kevin's method requires several calculations before we arrive at the equation to solve.

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