Question 1

(a) Factorise fully $9a^2 - 6ab + 12ac - 8bc$.

$$9a^2 - 6ab + 12ac - 8bc = 3a(3a - 2b) + 4c(3a - 2b)$$
$$= (3a - 2b)(3a + 4c).$$

(b) Factorise $9x^2 - 16y^2$.

$$9x^2 - 16y^2 = (3x - 4y)(3x + 4y).$$
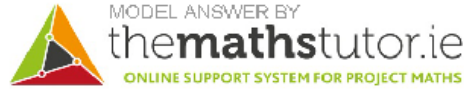
(c) Use factors to simplify the following: $\dfrac{2x^2 + 4x}{2x^2 + x - 6}$.

$$\frac{2x^2 + 4x}{2x^2 + x - 6} = \frac{2x(x + 2)}{(x + 2)(2x - 3)}$$
$$= \frac{2x}{2x - 3}.$$

(a) Factorise $5x - 15$ and $6 - 2x$.

$$5x - 15 = 5(x - 3) \qquad \text{and} \qquad 6 - 2x = 2(3 - x) = -2(x - 3)$$
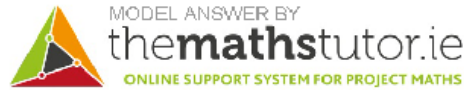
If A and B are variable quantities, we say that A is proportional to B if the fraction $\dfrac{A}{B}$ is a constant.

(b) Using your answers to part **(a)** above, show that $5x - 15$ is proportional to $6 - 2x$.

$$\frac{5x - 15}{6 - 2x} = \frac{5(x - 3)}{-2(x - 3)} = -\frac{5}{2}$$

This fraction is a constant, so $5x - 15$ is indeed proportional to $6 - 2x$.

(c) Is $x^2 + 3x + 2$ proportional to $2x + 2$? Justify your answer.

Firstly, we need to factorise our two quantities:

$$x^2 + 3x + 2 = (x + 2)(x + 1) \qquad \text{and} \qquad 2x + 2 = 2(x + 1)$$

Now, we divide one by the other:

$$\frac{x^2 + 3x + 2}{2x + 2} = \frac{(x + 2)(x + 1)}{2(x + 1)} = \frac{x + 2}{2}$$

This fraction is **not** constant since it depends on x. Thus, the two quantities are not proportional.