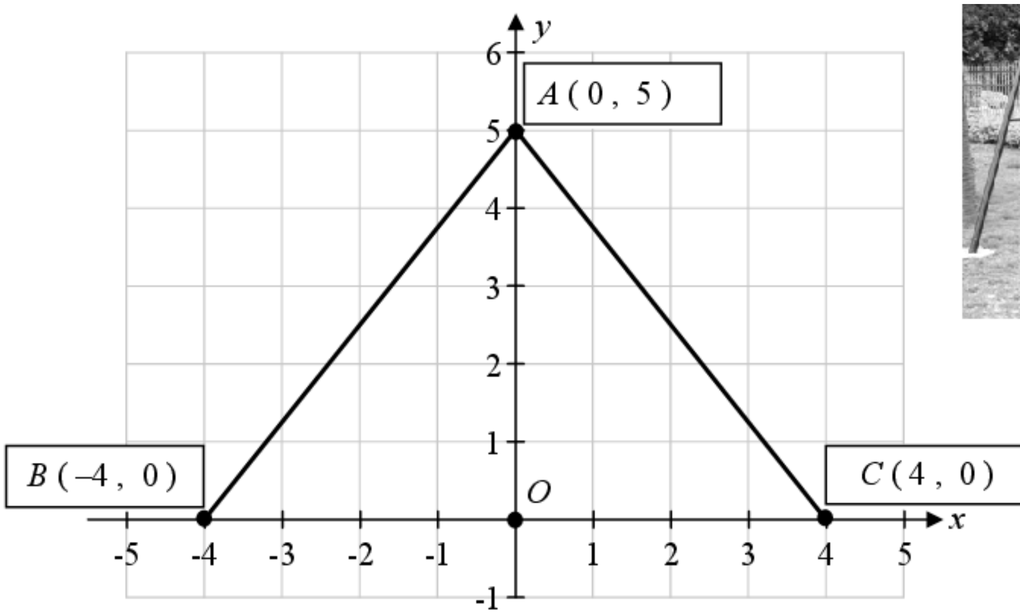
Question 1



(i) Write the co-ordinates of the points A, B, and C in the spaces provided in the diagram.

(ii) Find the total length of metal bar needed to make this part of the swing.
Give your answer in metres, correct to one decimal place.

$|AB| = \sqrt{4^2 + 5^2} = \sqrt{41}$.

Similarly, $|AC| = \sqrt{41}$.

Total length of metal bar $= 2\sqrt{41} = 12\cdot80... = 12\cdot8$ m, correct to one decimal place.

(iii) Find the slope of AB and the slope of AC.

AB: Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{5}{4}$ or $1\cdot25$.	*AC*: Slope $= \dfrac{5-0}{0-4} = -\dfrac{5}{4}$ or $-1\cdot25$.

(iv) Is AB perpendicular to AC? Give a reason for your answer.

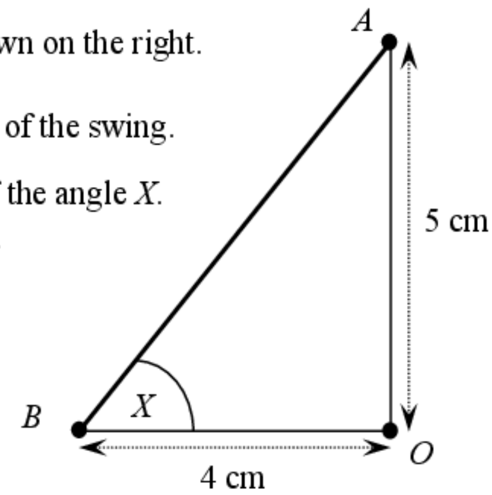
Answer: No

Reason: Product of slopes $= \dfrac{5}{4} \times -\dfrac{5}{4} = -\dfrac{25}{16} \neq -1$.

Or: Reason: When you invert one slope and change the sign, you don't get the other slope.

(v) Madison draws the scale diagram of the triangle *OAB* shown on the right.
She marks in the angle X.
Recall that $[AB]$ is a metal bar, which is part of the frame of the swing.

Write down the value of $\tan X$, and hence find the size of the angle X.
Give the size of the angle X correct to two decimal places.



$\tan X = \dfrac{5}{4}$
$\lvert\angle X\rvert = \tan^{-1}\left(\dfrac{5}{4}\right) = 51\cdot 340... = 51\cdot 34°$, correct to two decimal places.

In order to increase the height of the swing, it is decided to increase X by 20%.
The distance $\lvert AB\rvert$ will be kept the same.

(vi) Find the new height of the swing. Give your answer in metres, correct to one decimal place.

Recall from **(ii)** that $\lvert AB\rvert = \sqrt{41}$ m.

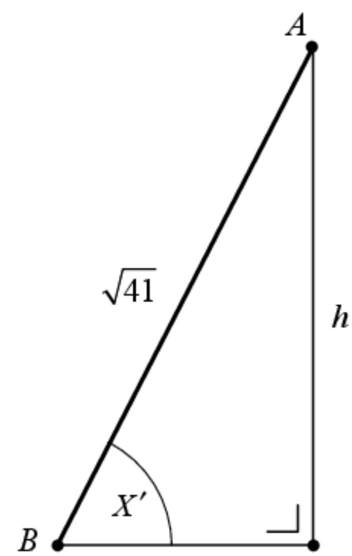
Increase $\lvert\angle X\rvert$ by 20% to get $\lvert\angle X'\rvert$:

$$\lvert\angle X'\rvert = 51\cdot 34 \times 1\cdot 2 = 61\cdot 608°$$

From the diagram, $\sin X' = \sin 61\cdot 608 = \dfrac{h}{\sqrt{41}}$.

$$\Rightarrow \quad h = \sqrt{41}\sin 61\cdot 608 = 5\cdot 632... = 5\cdot 6 \text{ m},$$

correct to one decimal place.



Question 2

(i) Find the slope of the line l.

Method 1:	*Method 2*:
$-3y = -x + 6$ *Step 1* $3y = x - 6$ $y = \frac{1}{3}x - 2$ *Step 2* $\Rightarrow$ Slope $= \frac{1}{3}$ *Step 3*	Slope $= -\frac{a}{b}$ $= -\frac{1}{-3}$ $= \frac{1}{3}$

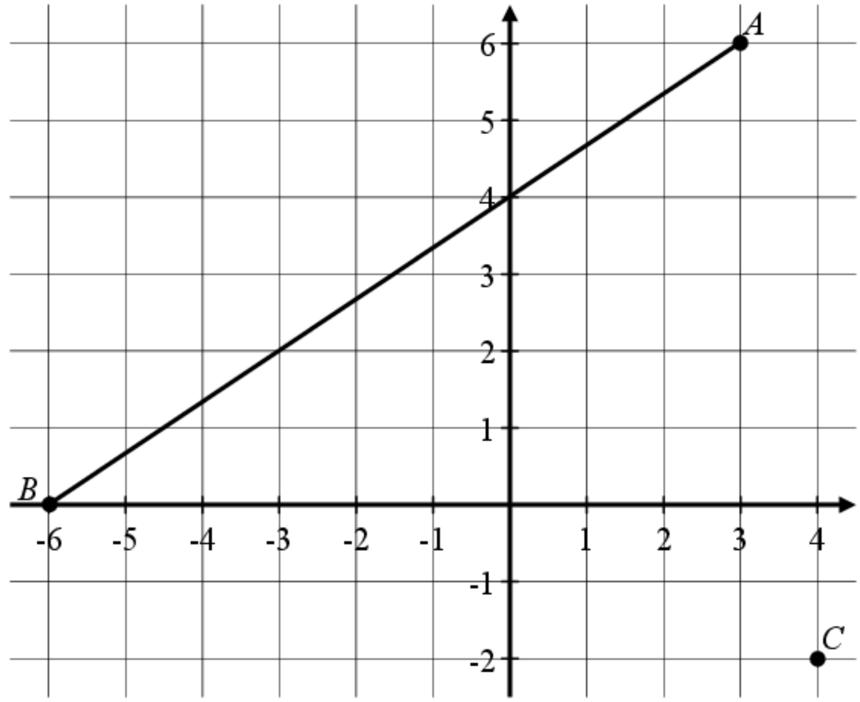
(ii) Show that the point $(1,-2)$ is **not** on the line l.

Sub in $(1,-2)$ to l: LHS $= 1 - 3(-2) - 6 = 1 \neq 0 =$ RHS. Point not on l.

(iii) The line k passes through $(1,-2)$ and is parallel to the line l.
Find the equation of the line k.

Slope of $k = \frac{1}{3}$.	Point on $k = (1,-2)$.
Equation of k: $y - (-2) = \frac{1}{3}(x - 1)$ $\Rightarrow \quad y = \frac{x}{3} - \frac{7}{3}$ *or* $\quad x - 3y - 7 = 0$	*Or*: Equation of k: $x - 3y + c = 0$ $\Rightarrow \quad 1 - 3(-2) + c = 0$ $\Rightarrow \quad c = -7$ $\Rightarrow \quad x - 3y - 7 = 0$

Question 3



(a) Write the coordinates of A, B and C.

$A(\ 3\ ,\ 6\ )\quad B(-6\ ,\ 0)\quad C(\ 4\ ,-2\ )$

(b) Find the co-ordinates of D, the mid-point of [AB].

$$D = \left(\frac{3-6}{2}, \frac{6+0}{2}\right) = \left(-\frac{3}{2}, 3\right)$$

(c) Find the equation of the line AB.

$$\text{Slope } AB = \frac{0-6}{-6-3} = \frac{2}{3}$$

Equation AB: $y - 0 = \frac{2}{3}(x+6)$ **or** $y - 6 = \frac{2}{3}(x-3)$

or

$$y = \frac{2}{3}x + 4$$

$$2x - 3y + 12 = 0$$

(d) Find the equation of the line through C, perpendicular to AB.

Perpendicular slope $= -\dfrac{3}{2}$

Line through C: $y+2 = -\dfrac{3}{2}(x-4)$

$3x + 2y - 8 = 0$

or

The line is of the form $3x + 2y + c = 0$

$(4, -2)$: $3(4) + 2(-2) + c = 0 \Rightarrow c = -8$

$3x + 2y - 8 = 0$

(e) Let E be the point where this perpendicular line through C intersects AB.
Calculate the coordinates of the point E.

E the point of intersection of two lines

$2x - 3y + 12 = 0$ (i)

$3x + 2y - 8 = 0$ (ii)

$2\times$(i) $4x - 6y = -24$

$+\ 3\times$(ii) $9x + 6y = 24$

or

$y = \dfrac{2x+12}{3}$

$\Rightarrow 3x + 2\left(\dfrac{2x+12}{3}\right) - 8 = 0$

$\Rightarrow 9x + 4x + 24 - 24 = 0$

$\Rightarrow$ $x = 0$ and $y = 4$

(f) Which is the shorter distance, $|CD|$ or $|CE|$? Find this distance.

$|CD| = \sqrt{\left(4+\dfrac{3}{2}\right)^2 + (-2-3)^2} = \sqrt{55\cdot 25}$ or $7{\cdot}433$

$|CE| = \sqrt{(4-0)^2 + (-2-4)^2} = \sqrt{52}$ or $7{\cdot}211$

$|CE|$ is the shorter distance

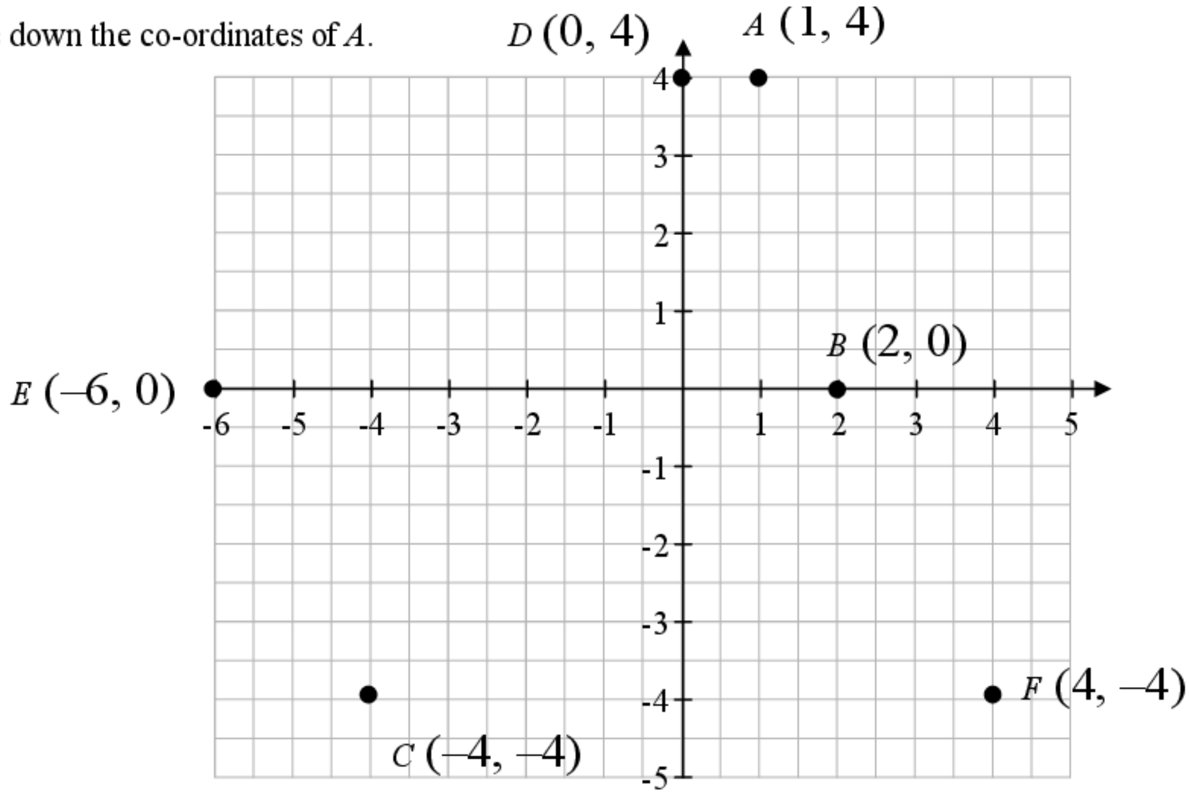
or

$|CE|$ (is the perpendicular distance and therefore is the shorter distance.)

$|CE| = \sqrt{(4-0)^2 + (-2-4)^2} = \sqrt{52}$ or $7{\cdot}211$

Question 4

(a) Write down the co-ordinates of A.



(b) Plot the following points on the diagram above.

B	***C***	***D***	***E***	***F***
(2, 0)	(– 4, – 4)	(0, 4)	(– 6, 0)	(4, – 4)

(c) Calculate the midpoint of [DF].

$$\left(\frac{0+4}{2},\frac{4-4}{2}\right)=(2,0)$$

(d) Find the slope of BF.

$$\frac{-4-0}{4-2}=\frac{-4}{2}-2$$

(e) Write down the equation of the line BF in the form $y = mx + c$.

$y=-2x+4$

$y - 0 = -2(x - 2)$
$y = -2x + 4$

(f) Find the slope of the line CE.

$$\frac{0-(-4)}{-6-(-4)} = \frac{4}{-2} = -2$$

(g) Write the equation of the line CE in the form of $ax + by + c = 0$.

$y - 0 = -2(x+6)$

$y = -2x - 12$

$2x + y + 12 = 0$

(h) What is the ratio of the area of the triangle BCE to the area of the triangle BCF?

Area of ΔBCE

$\frac{1}{2}(8)(4)$

16

Area of ΔBCF

$\frac{1}{2}(8)(4)$

16

Ratio: 1:1 or 1/1

(i) State whether the two triangles in part **(h)** above are congruent.
Give a reason for your answer.

Answer: yes

Reason: $CFBE$ is a parallelogram and CB is a diagonal which divides the parallelogram into two congruent triangles.

Or

SSS or SAS or ASA argument

Question 5

(a) Which line has the greatest slope? Give a reason for your answer.

Line 3 OR $y = 5x + 20$

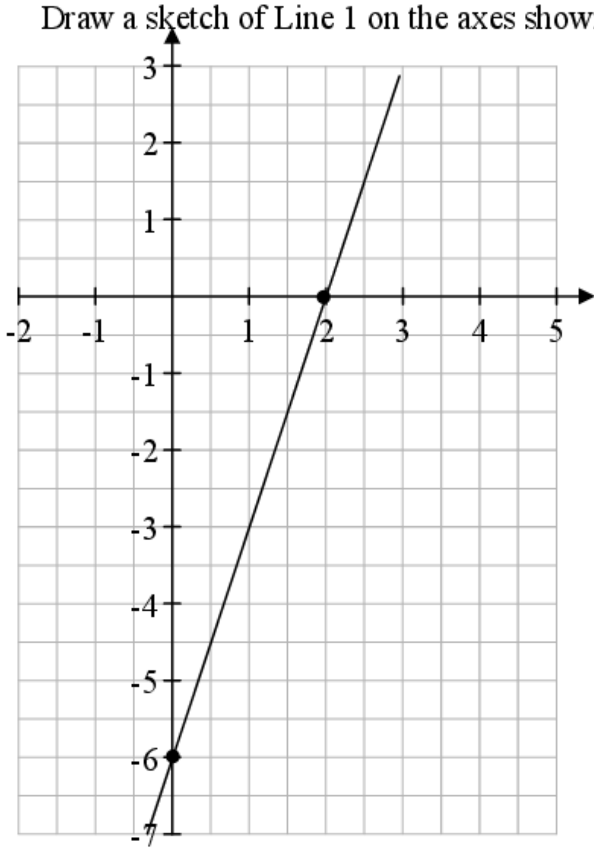
5 is the biggest number in front of x for any of the lines

(b) Which lines are parallel? Give a reason for your answer.

Line 1 and Line 2
$y = 3x - 6$ and $y = 3x + 12$

They have the same slope (3)

(c) Draw a sketch of Line 1 on the axes shown.



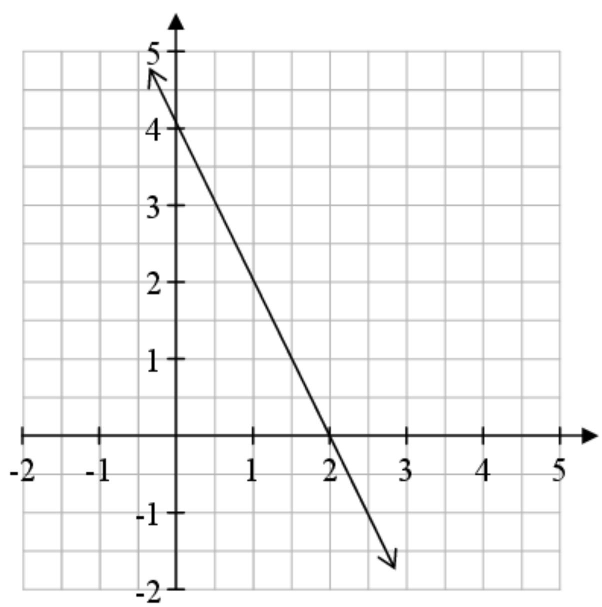
$y = 3x - 6$

$x = 0,\ y = -6$

$y = 0,\ x = 2$

$(2,0), (0,-6)$

(d) The diagram below represents one of the given lines. Which line does it represent?



$\text{slope} = \dfrac{0-4}{2-0} = -2$

y-intercept $= 4$

equation $y = -2x + 4$

Answer = Line ________5 $(y = -2x + 4)$

(e) The table shows some values of x and y for the equation of one of the lines.
Which equation do they satisfy?

x	y
7	12
9	20
10	24

$y = 4x - 16$
$y = 4(7) - 16 = 12$
$y = 4(9) - 16 = 20$
$y = 4(10) - 16 = 24$

$m = \dfrac{20-12}{9-7} = 4$
$y - 12 = 4(x - 7)$
$y = 4x - 16$

Answer = Line ______6______

(f) There is one value of x which will give the same value of y for Line 4 as it will for Line 6. Find, using algebra, this value of x and the corresponding value of y.

$y = x - 7$
$y = 4x - 16$
$0 = 3x - 9$
$x = 3$
$y = 3 - 7 = -4$

$(3, -4)$
Ans: $x = 3$

(g) Verify your answer to **(f)** above.

Line 4
$y = (3) - 7 = -4$
Line 6
$y = 4(3) - 16 = -4$