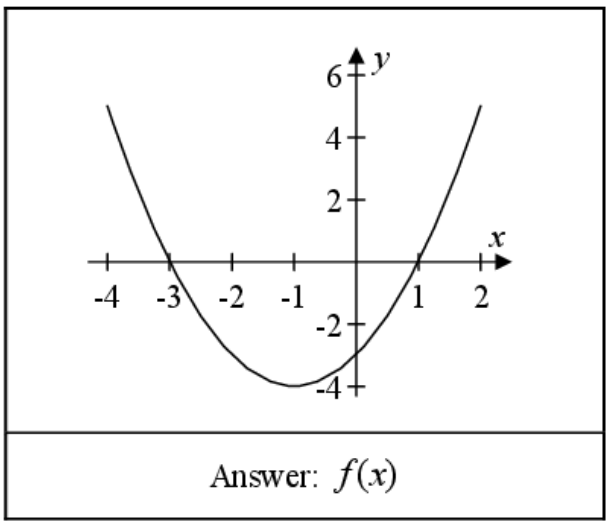
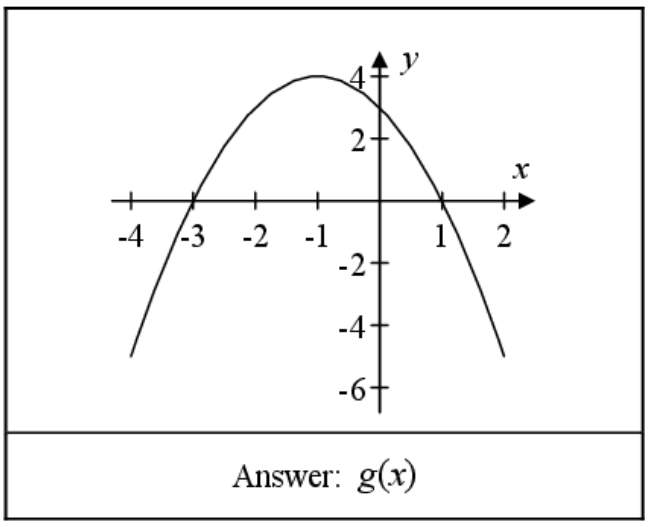
Question 1

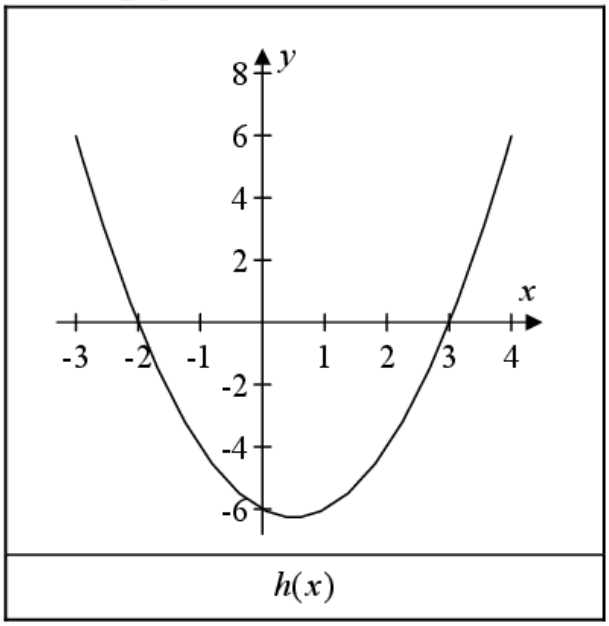


Answer: $f(x)$

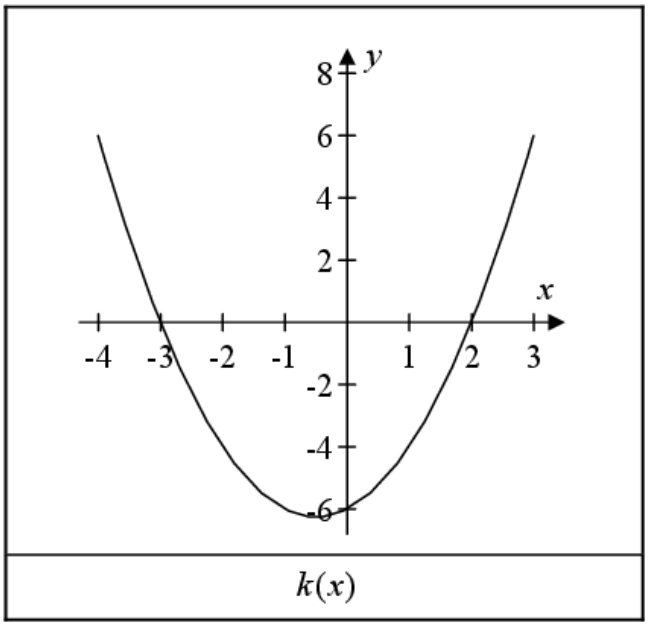


Answer: $g(x)$

(b) The graphs of the functions $y = h(x)$ and $y = k(x)$ are shown below.



$h(x)$



$k(x)$

Write down the roots of each function.
Hence, or otherwise, write down an equation for each function.

Roots of $h(x)$: $x = -2$ and $x = 3$.

Equation: $h(x) = (x+2)(x-3)$, or $h(x) = x^2 - x - 6$.

[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $h(0) = -6$, which corresponds to the graph.]

Roots of $k(x)$: $x = -3$ and $x = 2$.

Equation: $k(x) = (x+3)(x-2)$, or $k(x) = x^2 + x - 6$.

[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $k(0) = -6$, which corresponds to the graph.].

Question 2

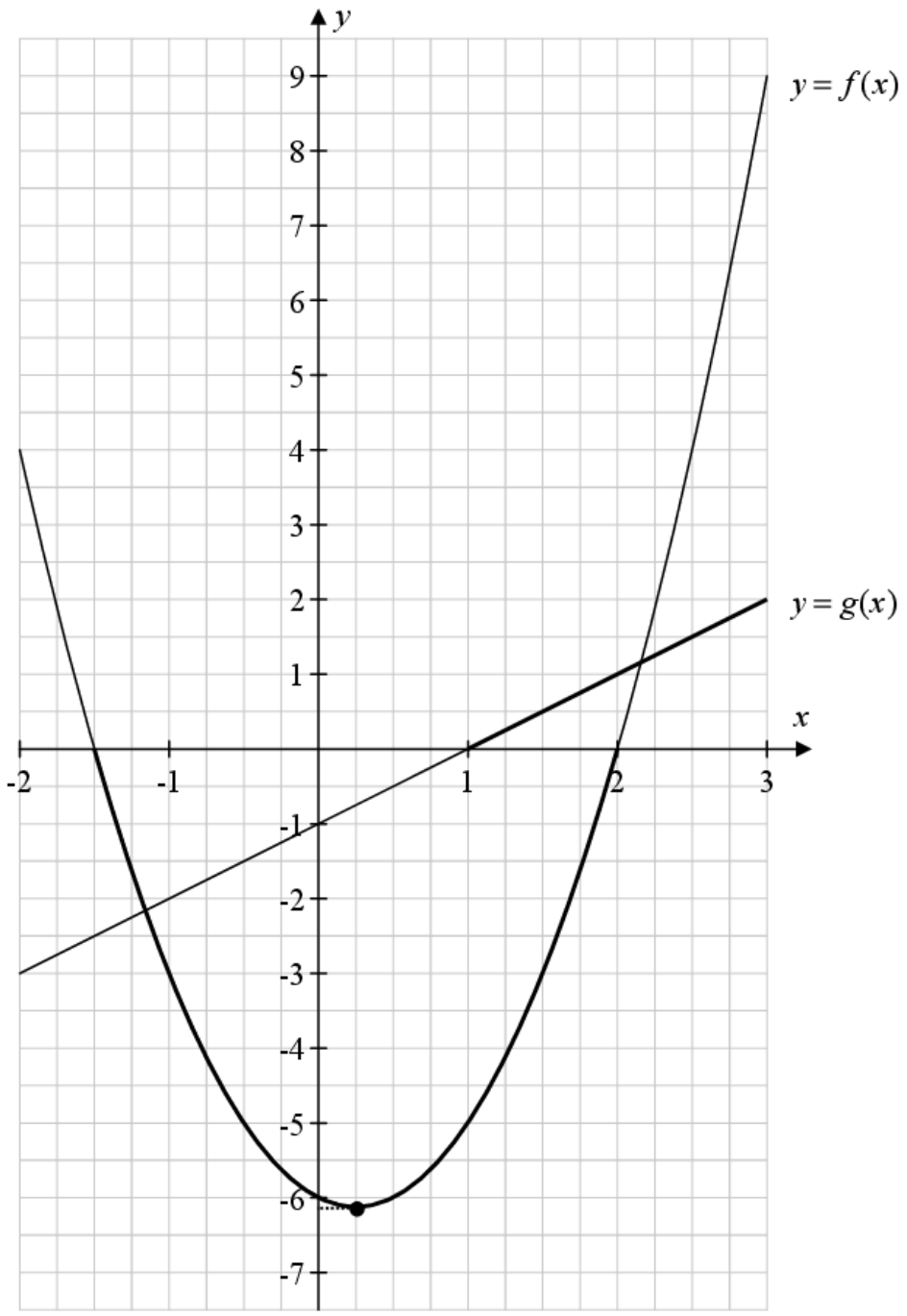
(i) g is the function $g: x \mapsto x-1$, where $x \in \mathbb{R}$. Find the value of each of the following.

$g(3) = 3 - 1 = 2.$

$g(-2) = -2 - 1 = -3.$

(ii) f is the function $f: x \mapsto 2x^2 - x - 6$, where $x \in \mathbb{R}$.

Using the same axes and scales, draw the graphs of the functions $y = f(x)$ and $y = g(x)$ in the domain $-2 \leq x \leq 3$.



Graphing g:

Straight line, so only need the two points from **(i)**:

$(3,2)$ and $(-2,-3)$.

Or:

$g(x) = x - 1$

x	x	- 1	y
-2	-2	- 1	-3
-1	-1	- 1	-2
0	0	- 1	-1
1	1	- 1	0
2	2	- 1	1
3	3	- 1	2

Graphing f:

$f(-2) = 4$

$f(-1) = -3$

$f(0) = -6$

$f(1) = -5$

$f(2) = 0$

$f(3) = 9$

Or:

$f(x) = 2x^2 - x - 6$

x	$2x^2$	- x	- 6	y
-2	8	+2	- 6	4
-1	2	+ 1	- 6	- 3
0	0	0	- 6	- 6
1	2	- 1	- 6	- 5
2	8	- 2	- 6	0
3	18	- 3	- 6	9

Use your graphs from **(ii)** to estimate:

(iii) the minimum value of $f(x)$

$f_{\min}(x) = -6 \cdot 1$ ... *see graph*

(iv) the range of values of x for which $f(x) < 0$

$-1 \cdot 5 < x < 2$ ... *see graph*

(v) the range of values of x for which $g(x) \geq 0$.

$x \geq 1$ ... *see graph*

Question 3

$2a - b + 2c$		
$8a - 2b + 2c$	Diff $= 6a - b$	
$18a - 3b + 2c$	Diff $= 10a - b$	Diff $= 4a$
$32a - 4b + 2c$	Diff $= 14a - b$	Diff $= 4a$
$50a - 5b + 2c$	Diff $= 18a - b$	Diff $= 4a$

2^{nd} difference is constant therefore the relationship is quadratic

Question 4

Solve $f(x) = 0$	Solve $g(x) = 0$	Solve $h(x) = 0$
$(2x - 3)(x + 2) = 0$ $x = \frac{3}{2}, \quad x = -2$	$(x - 3)(x - 3) = 0$ $x = 3$	$x(x - 2) = 0$ $x = 0, \quad x = 2$

Diagram 1

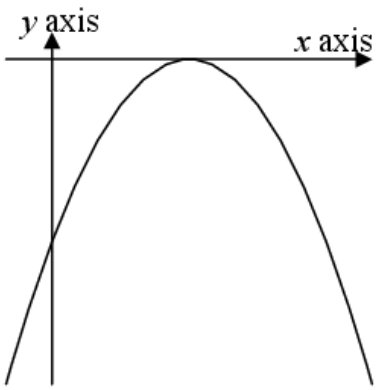
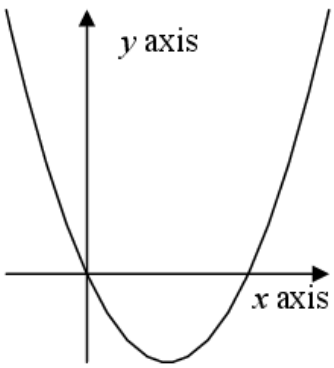
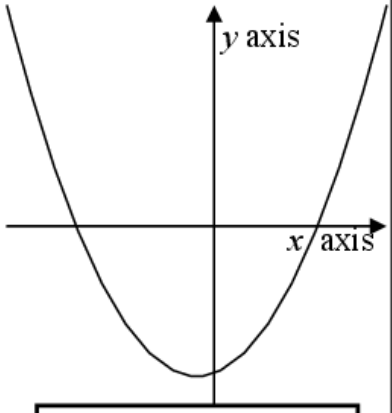


Diagram 2



$h(x)$

Diagram 3



$f(x)$,

Diagram 4

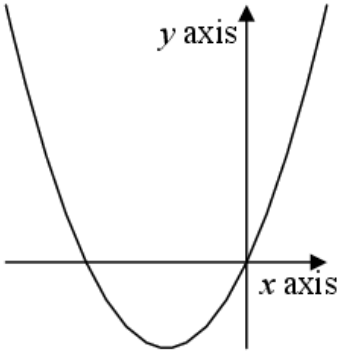
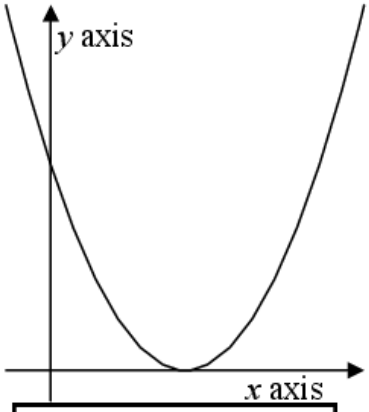
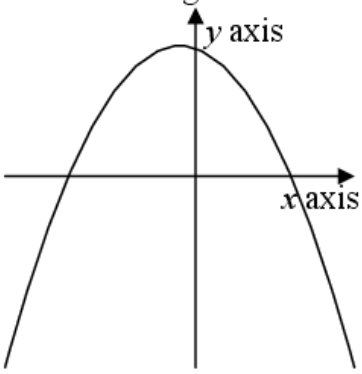


Diagram 5



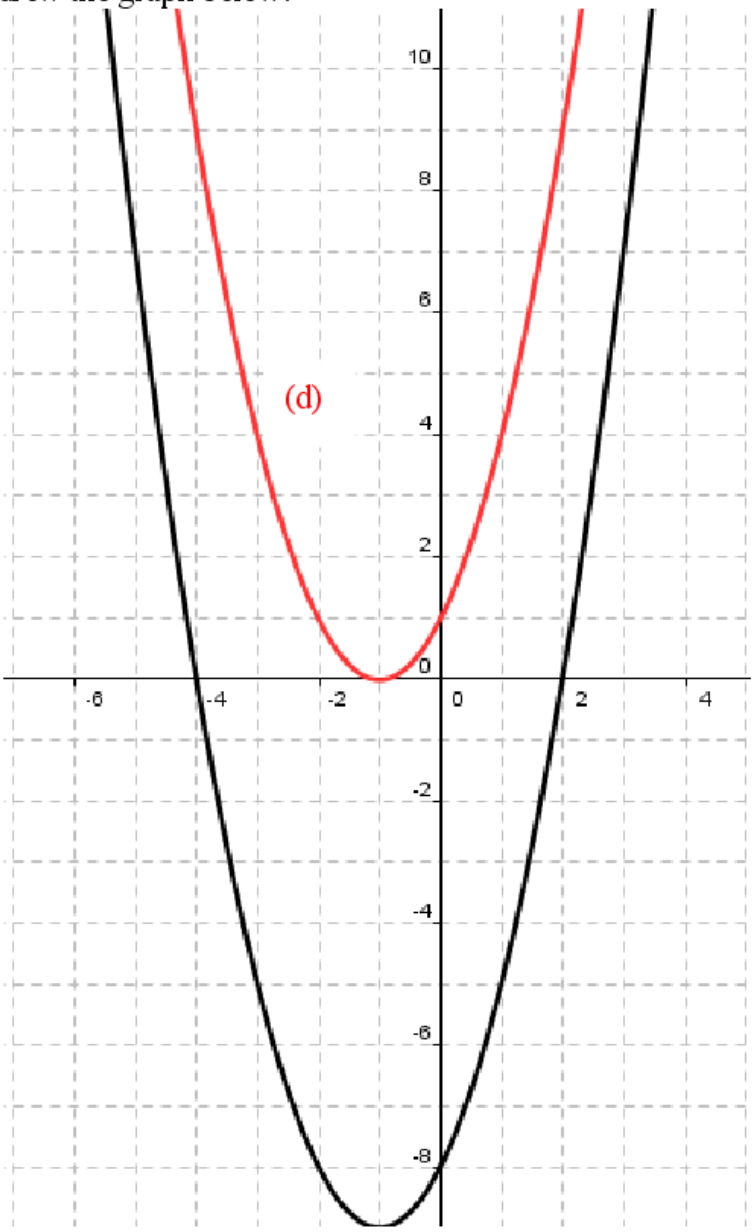
$g(x)$

Diagram 6



Question 5

A group of four students is studying graphs of functions of the form $f : x \mapsto x^2 + 2x + k, \quad x \in \mathbb{R}$.
Each takes an integer value of k and draws the graph of their function in a suitable domain.
Maria took $k = -8$ and drew the graph below.



(a) Use the graph to write down the roots of the equation $x^2 + 2x - 8 = 0$.

Roots 2 and -4

(b) Keith's graph passes through the point (3, 2). Find the value of k that Keith used.

$$f(x) = x^2 + 2x + k$$
$$2 = 3^2 + 2(3) + k$$
$$2 = 9 + 6 + k$$
$$k = -13$$

(c) On Alice's graph, the two roots of the function are the same.
Find the value of k that Alice used.

$$(x+t)^2 = x^2 + 2x + k$$
$$x^2 + 2tx + t^2 = x^2 + 2x + k$$
$$\Rightarrow 2t = 2$$
$$t = 1$$

(d) Draw a sketch of Alice's function on the diagram shown in part **(a)**.

(e) Emma's graph shows that the roots of her function are – 5 and 3.
Find the value of k that she used.

$$(x+5)(x-3) = 0$$
$$x^2 + 2x - 15 = 0$$
$$\Rightarrow k = -15$$

Constant is product of roots

$$-5 \times 3 = -15$$
$$\Rightarrow k = -15$$