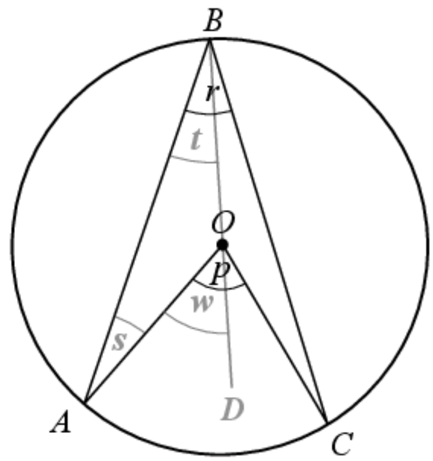
Question 1

(a) Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Diagram:



Given: A circle with centre O. Points A, B, and C on the circle. Angles p and r, as shown.

To Prove: $p = 2r$.

Construction: Join B to O, and extend to D. Mark the angles s, t, and w.

Proof:

$\lvert OA\rvert = \lvert OB\rvert$	radii of circle	*Step 1*
$\therefore s = t$	isosceles triangle	*Step 2*
$w = s + t$	exterior angle	*Step 3*
$\therefore w = 2t$		*Step 4*

Similarly, $(p - w) = 2(r - t)$.

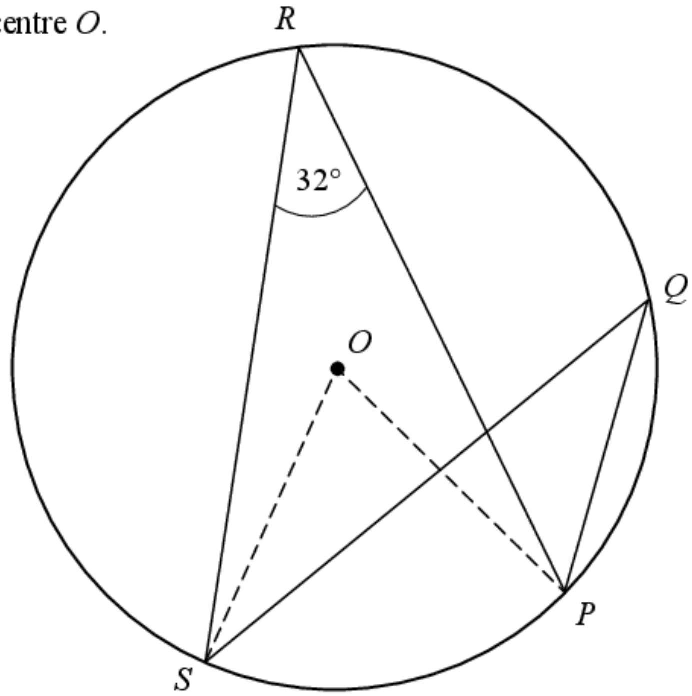
So $p = (p - w) + w$

$= 2(r - t) + 2t$

$= 2r$ *Step 5*

(b) P, Q, R, and S are points on a circle with centre O.

$|\angle PRS| = 32°$, as shown.



(i) Find $|\angle SOP|$.

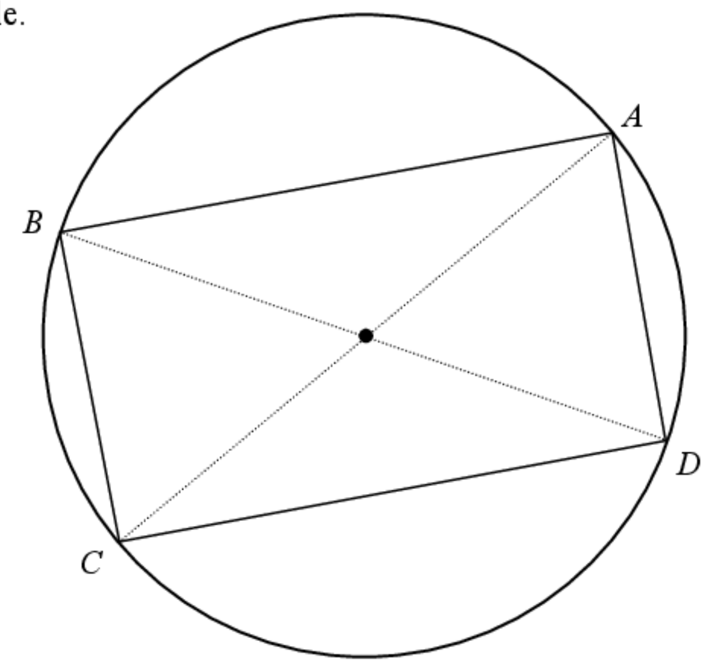
$$|\angle SOP| = 2 \times 32° = 64°.$$

(ii) Find $|\angle SQP|$.

$$|\angle SQP| = |\angle SRP| = 32°.$$

(c) A, B, C, and D are points on a circle, as shown below.
$[AC]$ and $[BD]$ are diameters of the circle.

Prove that $ABCD$ is a rectangle.



We just need to prove that the four angles are 90°.

$|\angle BAD| = |\angle BCD| = 90°$, as $[BD]$ is a diameter.

Similarly, $|\angle CBA| = |\angle CDA| = 90°$.

So $ABCD$ is a rectangle.

Question 2

(i) Prove that $\Delta\, MNP$ and $\Delta\, QRP$ are similar.

Proof:	$\lvert\angle MNP\rvert$	$=$	$\lvert\angle PRQ\rvert$	(given)
	$\lvert\angle NPM\rvert$	$=$	$\lvert\angle QPR\rvert$	(vertically opposite)
	$\lvert\angle NMP\rvert$	$=$	$\lvert\angle PQR\rvert$	(third angle)
	$\Rightarrow$ Triangles are similar.			

(ii) Is NM parallel to QR? Give a reason for your answer.

Answer: Yes

Reason: $\lvert\angle MNP\rvert = \lvert\angle PRQ\rvert$ *or* $\lvert\angle NMP\rvert = \lvert\angle PQR\rvert$ *or* alternate angles are equal.

Given $|MN| = 6,\ |NP| = 4,\ |QP| = 9,$ and $|PR| = 10,$ find:

(iii) $|QR|$

By similar triangles $\triangle MNP$ and $\triangle QRP$:

$$\frac{|QR|}{6} = \frac{10}{4}$$

$$\Rightarrow |QR| = 6\times\frac{10}{4} = 15.$$

(iv) $|QM|$.

By similar triangles $\triangle MNP$ and $\triangle QRP$:

$$\frac{|PM|}{9} = \frac{6}{15} \text{ or } \frac{4}{10}$$

$$\Rightarrow |PM| = \frac{18}{5} \text{ or } 3\cdot 6$$

$$\Rightarrow |QM| = 9 + 3\cdot 6 = \frac{63}{5} \text{ or } 12\cdot 6.$$

Or:

$$\frac{|PM|}{4} = \frac{9}{10}$$

$$\Rightarrow |PM| = 4\times\frac{9}{10} = \frac{18}{5} \text{ or } 3\cdot 6$$

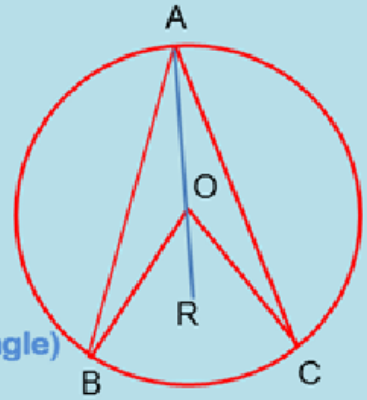
$$\Rightarrow |QM| = 9 + 3\cdot 6 = \frac{63}{5} \text{ or } 12\cdot 6.$$

Question 3

Given: A circle with centre O, with points A, B and C on the circle

To Prove: $|\angle BOC| = 2|\angle BAC|$

Construction: Join A to O and extend to R



Proof: In the triangle AOB

$|AO| = |OB|$ Radii

$\Rightarrow |\angle OBA| = |\angle OAB|$ Theorem 2 (isosceles Δ)

$|\angle BOR| = |\angle OBA| + |\angle OAB|$ Theorem 6 (exterior angle)

$\therefore \; |\angle BOR| = |\angle OAB| + |\angle OAB|$

$\therefore \; |\angle BOR| = 2|\angle OAB|$

Similarly $|\angle ROC| = 2|\angle OAC|$

$\therefore \; |\angle BOC| = 2|\angle BAC|$