Question 1

Question 2 **(Suggested maximum time: 5 minutes)**

The sets U, A, and B are defined as follows, where U is the universal set:

$$U = \{2,3,4,5,\ldots,30\}$$
$$A = \{\text{multiples of } 2\}$$
$$B = \{\text{multiples of } 3\}$$
$$C = \{\text{multiples of } 5\}$$

(a) Find $\#[(A \cup B \cup C)']$ the number of elements in the complement of the set $A \cup B \cup C$.

The elements in the set $(A \cup B \cup C)'$ will be the whole numbers between 2 and 30 inclusive which are **not** multiples of 2, 3 or 5. These numbers are $\{7,11,13,17,19,23,29\}$. This means that the number of elements $\#[(A \cup B \cup C)'] = 7$.

(b) How many divisors does each of the numbers in $(A \cup B \cup C)'$ have?

Each of these numbers has exactly two divisors: itself and 1.

(c) What name is given to numbers that have exactly this many divisors?

These numbers are called **prime** numbers.

Question 6 **(Suggested maximum time: 5 minutes)**

Niamh is in a clothes shop and has a voucher which she **must** use. The voucher gives a €10 reduction when buying goods to the value of at least €35. She also has €50 cash.

(a) Write down an inequality in x to show the range of cash that she could spend in the shop.

The voucher requires a minimum of €35 to be used. Since Niamh must use her voucher, she must spend a minimum of €35. If she spends this amount, she gets a €10 discount, meaning the minimum she can spend is €25. The maximum amount of cash she can spend is €50, so the range will be $25 \leqslant x \leqslant 50$.

(b) Niamh buys one item of clothing in the shop, using the voucher as she does so. Write an inequality in y to show the range of possible prices that this item could have, before the €10 reduction is applied.

As before, the minimum value Niamh can spend is €35. The maximum amount of cash she can spend is €50, and including the discount, the item of clothing could cost up to €60. Thus the range will be $35 \leqslant y \leqslant 60$.

Question 3

(a) Write down the next 3 natural numbers, in terms of n.

The next 3 natural numbers are $n+1$, $n+2$ and $n+3$.

Hence, or otherwise, complete the following.

(b) Show that the sum of any 3 consecutive natural numbers is divisible by 3.

Consider 3 consecutive natural numbers n, $n+1$ and $n+2$. If we sum these numbers, we get $n+(n+1)+(n+2)=3n+3$. We can factorise this sum into $3(n+1)$, which is divisible by 3, concluding the proof.

(c) Prove or disprove the following statement: "The sum of any 4 consecutive natural numbers is never divisible by 4."

Consider 4 consecutive natural numbers n, $n+1$, $n+2$ and $n+3$. If we sum these numbers, we get $n+(n+1)+(n+2)+(n+3)=4n+6$. Now 4 divides the $4n$ term, but 4 does not divide 6. Therefore, regardless of the value of n, 4 cannot divide the sum, proving the statement.

Question 4

(a) Place the following numbers in order, starting with the smallest:

$$\frac{3}{2} \qquad 1{\cdot}4 \qquad \sqrt{2}$$

$1{\cdot}4,\ \sqrt{2},\ \frac{3}{2}.$ $\left(\sqrt{2} = 1{\cdot}414\ldots,\ \frac{3}{2} = 1{\cdot}5.\right)$

(b) Which one of the following is **not** a rational number? Explain your answer.

$$3\frac{1}{7} \qquad 3{\cdot}142 \qquad \frac{22}{7} \qquad \pi$$

Answer: π

Reason: It cannot be written as a fraction.

(c) **(i)** Find the values of $\frac{4n^2+1}{13}$, where $n \in \{17,\ 19,\ 21\}$.

n	$\frac{4n^2+1}{13}$
17	$\frac{4\times(17)^2+1}{13} = \frac{1157}{13} = 89$
19	$\frac{4\times(19)^2+1}{13} = \frac{1445}{13}$ or $111^2/_{13}$
21	$\frac{4\times(21)^2+1}{13} = \frac{1765}{13}$ or $135^{10}/_{13}$

(ii) State which **one** of your answers is a natural number, and explain your choice.

Answer: 89.

Reason: It is a positive whole number.

Question 5

Number/Set	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$(\mathbb{R}\backslash\mathbb{Q})$	$\mathbb{R}$
$\sqrt{5}$	**No**	**No**	**No**	**Yes**	**Yes**
8	**Yes**	**Yes**	**Yes**	**No**	**Yes**
-4	**No**	**Yes**	**Yes**	**No**	**Yes**
3½	**No**	**No**	**Yes**	**No**	**Yes**
$\frac{3\pi}{4}$	**No**	**No**	**No**	**Yes**	**Yes**

(ii) In the case of $\sqrt{5}$ explain your choice in relation to the set of Irrational numbers $(\mathbb{R}\backslash\mathbb{Q})$ (i.e. give a reason for writing either 'Yes' or 'No').

$\sqrt{5}$ cannot be written as a fraction

(b) Use the properties of surds to show that $\sqrt{98}-\sqrt{18}+\sqrt{2}$ simplifies to $5\sqrt{2}$.

$$7\sqrt{2}-3\sqrt{2}+\sqrt{2}=5\sqrt{2}$$

Question 6

(a) Give two reasons why $-7{\cdot}3$ is not a natural number.

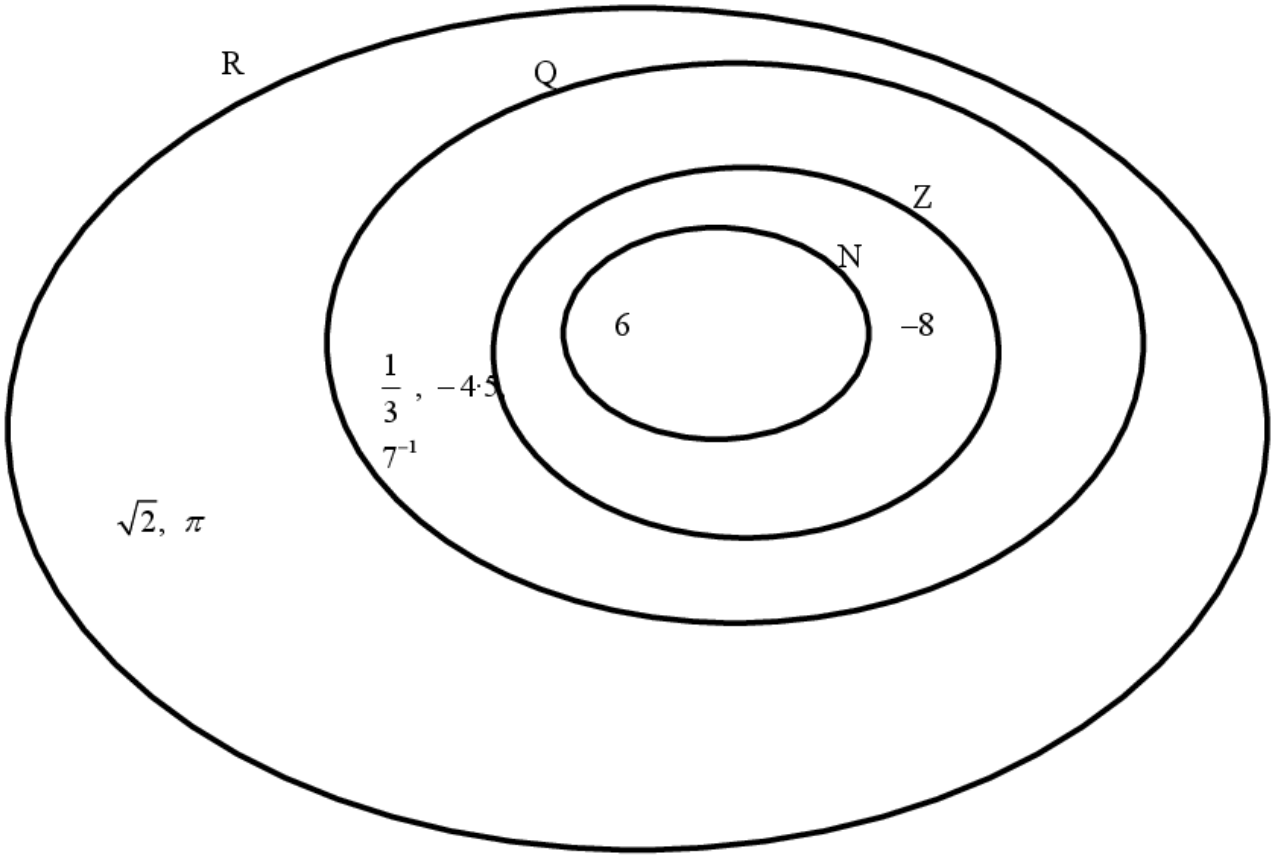
Reason 1:

$-7{\cdot}3$ is not a positive number OR $-7{\cdot}3$ is a minus number

Reason 2:

$-7{\cdot}3$ is not a whole number OR $-7{\cdot}3$ is a decimal

(b) The diagram represents the sets:
Natural Numbers N
Integers Z
Rational Numbers Q
Real Numbers R



Insert each of the following numbers in the correct place on the diagram:

$-8,\ \pi,\ \frac{1}{3}, 6, \sqrt{2},\ -4{\cdot}5$ and 7^{-1}.